Methods of determining weight scaling factors for geodetic–geophysical joint inversion

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A B S T R A C T

Geodetic–geophysical joint inversion is a hybrid inversion of different types of geodetic data, together with geophysical or seismic, geological data. In the joint inversion, weight scaling factors of different datasets are of vital importance and should be fixed properly. This paper aims to analyze the general weight scaling factor fixing methods and to study their impacts on joint inversion. The result, validated and evaluated by the cross validation test method, showed that it is not prudent to fix the inversion parameter only by considering the objective function to be a minimum and that the parameter should be determined by the actual circumstances. At last, a more reliable inversion result was obtained by using the Helmert method of variance components estimation (VCE) for the fixing of weight scaling factor.

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1. Introduction

Geodetic–geophysical joint inversion makes use of various data to extract common information, including geodetic, seismic, geological and geophysical data. The theory of geodetic–geophysical joint inversion is based on Backus–Gilbert theory (Backus and Gilbert, 1967, 1968, 1970). In 1977, Matsu’ura inverted for fault parameters by using geodetic data, and brought forward the concept of geodetic inversion (Matsu’ura, 1977a,b). Since then, a rapid and great development has occurred, with the continuum, linear form and single dataset replaced by the discrete, non-linear form and various datasets. Examples include the joint inversion of electronic distance-measuring instrument (EDM), global positioning system (GPS) and very long baseline interferometry (VLBI) (Lisowski et al., 1990), the joint inversion of leveling, GPS and gravity (Zhao and Sjöberg, 1993; Zhao, 1995; Li et al., 2002; Li and Xu, 2005), the joint inversion of geodetic and seismic data, and the joint inversion of geodetic, seismic and geologic data (Holt and Haines, 1993, 1995; Holt et al., 2000; Williams et al., 1993; Tinnon et al., 1995; Shen-Tu et al., 1998; Shen-Tu and Holt, 1999; Xu et al., 2000, 2003, 2005; Segall and Matthews, 1997; England and Molnar, 1997; Wu et al., 2001; Kreemer et al., 2000; Wan et al., 2004).

However, a problem in the geodetic–geophysical joint inversion remains: how should we fix weight scaling factors of different datasets so as to achieve a reliable result? As we know, different weight scaling factors represent different contributions to the inversion result, so only careful fixing of weight scaling factors can lead to a correct/sensible result. This paper firstly introduces the general methods to fix weight scaling factors in the geodetic–geophysical joint inversion, then discusses and analyzes them in detail on the basis of a case study in China.

2. Methods

This section introduces four methods that are usually used for fixing weight scaling factors. Without the loss of generality, the joint inversion of geodetic and seismic data is taken as a case to introduce the geodetic–geophysical joint inversion.

GPS data and seismic moment tensor data have been used to invert for the crustal motion velocity field and strain rates through bicubic Bessel interpolation (Shen-Tu et al., 1998; Holt et al., 2000). The relation between the horizontal velocity field \( u(x) \) and the rotation vector function \( W(x) \) can be described as (Haines and Holt, 1993)

\[
u(x) = \mathbf{r}[W(x) \times x]
\]
where \( \mathbf{x} \) is the unit radial position vector of points on the surface of the Earth and \( r \) is the Earth radius (6.371 x 10^6 mm). \( \mathbf{W}(\mathbf{x}) \) is expanded as a bicubic Bessel spline function of the longitude and latitude of the point. It can be constrained to simulate rigid plates and to accommodate rapid spatial variations in some regions where deforming is occurring. Eq. (1) can also be written as

\[
\mathbf{u}(\mathbf{x}) \equiv \frac{\mathbf{u}(\mathbf{x})}{r} = \mathbf{W}(\mathbf{x}) \times \mathbf{x}
\]

where the unit of \( \mathbf{u}(\mathbf{x}) \) in the equation is 1.0 x 10^{-9} a^{-1}, which is the same as that of \( \mathbf{W}(\mathbf{x}) \). The sets of equations that relate the rotation vector \( \mathbf{W}(\mathbf{x}) \) to the horizontal strain rates on the surface of a sphere are (Haines and Holt, 1993)

\[
\begin{align*}
\frac{\partial \mathbf{W}}{\partial \phi} &= \frac{\hat{\mathbf{Q}}}{\cos \theta} \mathbf{a}, \\
\frac{\partial \mathbf{W}}{\partial \theta} &= \hat{\mathbf{Q}} \cdot \frac{\partial \mathbf{W}}{\partial \phi}, \\
\frac{\partial \mathbf{W}}{\partial y} &= \frac{1}{2} \left( \hat{\mathbf{Q}} \cdot \frac{\partial \mathbf{W}}{\partial \theta} - \frac{\hat{\mathbf{Q}}}{\cos \theta} \frac{\partial \mathbf{W}}{\partial \phi} \right)
\end{align*}
\]

where the derivatives of \( \mathbf{W}(\mathbf{x}) \) are determined from the spline interpolation of the values of \( \mathbf{W}(\mathbf{x}) \) between the grid points and \( \hat{\mathbf{Q}} \) and \( \Theta \) are unit vectors in the east and north directions, respectively. The average seismic strain rates in Eqs. (3a)-(3c) can be obtained through a moment tensor summation (Kostrov, 1974). Often, the location of earthquakes is centralized in some regions, not uniformly distributed in the whole study area. So, in order to obtain the continuous strain rate filed in the whole study area, smoothing of each strain rate is needed (Haines and Holt, 1993). The corresponding objective function in the inversion is

\[
y = \lambda \sum_{i}^{N} \left( \epsilon_{ij}^{\text{fit}} - \epsilon_{ij}^{\text{obs}} \right)^{2} + (1 - \lambda) \sum_{j}^{M} \left( u_{ij}^{\text{fit}} - u_{ij}^{\text{obs}} \right)^{2}
\]

where subscripts \( ij, pq \) denote the tensor components of the strain rate tensor, \( Q_1 \) is the a priori variance–covariance matrix of the average horizontal strain rate tensor from the seismic moment tensor data, \( Q_2 \) is the a priori variance–covariance matrix of the velocity vectors, \( \epsilon_{ij}^{\text{obs}} \) is the observed average strain rate tensor by bicubic Bessel interpolation, \( u_{ij}^{\text{obs}} \) is the observed velocity field from GPS data, and \( u_{ij}^{\text{fit}} \) is the fitted velocity field through bicubic Bessel interpolation, \( M, N \) are the total numbers of GPS data and grid areas, respectively, and \( \lambda \) is the weight scaling factor of the seismic moment tensor data, and satisfies \( \lambda \in [0, 1] \). Here, it should be noted that \( Q_1 \) can be obtained in the process of calculating \( \epsilon_{ij}^{\text{obs}} \) (Haines and Holt, 1993), and \( Q_2 \) can be obtained in the process of solving for the velocity field. \( Q_1 \) and \( Q_2 \) not only make each of the terms in Eq. (4) dimensionless, but also reflect initial weights of each observed value in the geodetic–geophysical joint inversion. But these initial weights are not accurate for the geodetic–geophysical joint inversion, because these datasets involve their own systematic errors separately. This leads to that the a prioris of unit weight of the datasets cannot be fixed exactly. On the other hand, mis-scaling of variance–covariance matrices may make the joint inversion to be underdetermined or mixed-determined. In addition, the weights are related to the inversion norms, different norms require different weights. It is necessary to rescale \( Q_1 \) and \( Q_2 \) by adjusting \( \lambda \) in the geodetic–geophysical joint inversion.

Because the relation between the velocity field and the rotation vector is linear (Eq. (1)), and the strain rate can be linearly expressed by the first derivative of the rotation vector (Eqs. (3a)-(3c)), Eq. (4) can be denoted by the vectors:

\[
y = \lambda v_{1}^{1} P_{1} V_{1} + (1 - \lambda) v_{2}^{1} P_{2} V_{2}
\]

here \( V_{1} = \epsilon_{ij}^{\text{fit}} - \epsilon_{ij}^{\text{obs}}, V_{2} = u_{ij}^{\text{fit}} - u_{ij}^{\text{obs}} \), are the vectors of residual errors, \( P_{1} = Q_{1}^{-1}, P_{2} = Q_{2}^{-1} \), are the weight matrices.

The observation equations are

\[
\begin{align*}
V_{1} &= B_{1} \mathbf{X} - L_{1} \\
V_{2} &= B_{2} \mathbf{X} - L_{2}
\end{align*}
\]

where \( \mathbf{X} \) is the unknown rotation vector, \( B_{1} \) and \( B_{2} \) are the coefficient matrices, and \( L_{1} \) and \( L_{2} \) are the observation vectors, which are \( \epsilon_{ij}^{\text{obs}} \) and \( u_{ij}^{\text{obs}} \), respectively.

In order to make the objective function \( y \) minimum, we set its first derivative with respect to \( \mathbf{X} \) to zero:

\[
\frac{\partial y}{\partial \mathbf{X}} = 2 \lambda v_{1} P_{1} B_{1} + 2(1 - \lambda) v_{2} P_{2} B_{2} = 0
\]

It also can be written as

\[
\lambda B_{1}^{T} P_{1} V_{1} + (1 - \lambda) B_{2}^{T} P_{2} V_{2} = 0
\]

From Eqs. (6a), (6b) and (8), the normal equation in the geodetic–geophysical joint inversion can be obtained:

\[
\lambda B_{1}^{T} P_{1} B_{1} + (1 - \lambda) B_{2}^{T} P_{2} B_{2} = \lambda B_{1}^{T} P_{1} L_{1} + (1 - \lambda) B_{2}^{T} P_{2} L_{2}
\]

that shows that \( \mathbf{X} \) is related to \( \lambda \), and the relation between them is non-linear. So, if \( \lambda \) is given a value in its domain, which is from 0 to 1, \( \mathbf{X} \) can be solved correspondingly. It is very crucial to fix the value of \( \lambda \) properly in the geodetic–geophysical joint inversion. Here, we discuss four methods for fixing \( \lambda \).

2.1. Method I: taking \( \lambda \) and \( \mathbf{X} \) as unknown parameters to invert together

Due to the fact that the relation between \( \mathbf{X} \) and \( \lambda \) is non-linear (Eq. (9)), and \( y \) is related to \( \lambda \) and \( \mathbf{X} \) (Eqs. (5), (6a) and (6b)), the relation between \( \lambda \) and \( y \) should also be non-linear and \( y \) can be decided by \( \lambda \). In a practical sense, a series of values of the objective function can be obtained when \( \lambda \) traverses its domain, and then the minimum of these values can be chosen as the final value of \( \lambda \). Accordingly, the unknown rotation vector \( \mathbf{X} \) and the scaling factor \( \lambda \) are taken as the final results.

2.2. Method II: fixing \( \lambda \) directly according to the a priori information

In order to scale the amplitudes of each data set, a ratio of the \( L_{2} \) norms of the observation vectors was used as the weight factor for joint inversion of body- and surface-wave data (Honda and Seno, 1989), tsunami and leveling data (Satake, 1993) and gravity and GPS data (Zhao, 1995). The weight scaling factor can be solved by

\[
1 - \lambda \frac{\lambda}{\lambda} = \frac{\|e_{ab}\|}{\|e_{ab}\|} \quad \text{or} \quad 1 - \lambda \frac{\lambda}{\lambda} = \frac{\sigma_{01}^{2}}{\sigma_{02}^{2}}
\]

where \( \sigma_{01}^{2} \) and \( \sigma_{02}^{2} \) are the squares of the standard errors of the observed average strain rate data, which is related to the seismic moment tensor, and the GPS velocity field data, respectively.
Fig. 1. Distribution of plotted grids (blue lines), focal mechanisms of all the earthquakes (1903–2003), GPS sites (black stars) and geological background (white lines) used in the study.

Table 1
Inverted scale factors, values of objective function and standard errors of unit weight

<table>
<thead>
<tr>
<th>Method</th>
<th>$\lambda$</th>
<th>$y$</th>
<th>$\delta_{01}$ (×10$^{-9}$ a$^{-1}$)</th>
<th>$\delta_{02}$ (×10$^{-9}$ a$^{-1}$)</th>
<th>$\delta_{03}$ (×10$^{-9}$ a$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.000</td>
<td>251.849</td>
<td>2.204</td>
<td>0.250</td>
<td>0.335</td>
</tr>
<tr>
<td>II</td>
<td>0.016</td>
<td>7299.655</td>
<td>0.691</td>
<td>1.991</td>
<td>1.803</td>
</tr>
<tr>
<td>III</td>
<td>0.671</td>
<td>3052.550</td>
<td>1.162</td>
<td>1.167</td>
<td>1.166</td>
</tr>
<tr>
<td>IV</td>
<td>N/A</td>
<td>4301.620</td>
<td>1.111</td>
<td>1.425</td>
<td>1.384</td>
</tr>
</tbody>
</table>

In this table, $\lambda$ is the scaling factor of the seismic moment tensor data; $y$ is the value of the objective function; $\delta_{01}$, $\delta_{02}$ and $\delta_{03}$ are the a posteriori standard errors of the unit weight of seismic data, the GPS data and the joint data, respectively; (I) represents the results when the value of the objective function is minimum; (II) represents the results when the scaling factor is fixed to be the ratio of standard errors of seismic data and GPS data; (III) represents the results by using VCE; (IV) represents the results when the scaling factors of the two datasets are equal.


denote the $L_2$ norm of the observed average strain rate data and the GPS velocity field data, respectively.

According to Eq. (10), $\lambda$ can be solved, and then the unknown rotation vector $\hat{X}$ can be got by Eq. (9).

2.3. Method III: solving $\lambda$ by Helmert method of variance components estimation

The uniform error equation can be written as Eqs. (6a) and (6b), regardless of the linearity between the geodetic observations $\mathbf{L}$ and the unknown parameter $\hat{\mathbf{X}}$. A joint inversion problem, shown as Eqs. (6a) and (6b), can be solved by using variance components estimation (VCE) to fix the scaling factor. In this paper, we assume the initial variances of unit weight of seismic moment tensor data and GPS data to be $\sigma_{01}^2$ and $\sigma_{02}^2$, respectively. The relation between their estimates and the corresponding sums of squared misfit is (Cui et al., 2001; Grafarend, 2006):

$$ S_{VCE} = W_0 $$

and

$$ S = \left[ n_1 - 2\text{tr}(N^{-1}N_1) + \text{tr}(N^{-1}N_1)^2 \right] \left[ \frac{\text{tr}(N^{-1}N_1N^{-1}N_2)}{n_2 - 2\text{tr}(N^{-1}N_2) + \text{tr}(N^{-1}N_2)^2} \right] $$

Table 2
Comparisons of inverted velocity field

<table>
<thead>
<tr>
<th>Method</th>
<th>Ave. (cm/a)</th>
<th>Ave. (cm/a)</th>
<th>Max. (cm/a)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$v^i$</td>
<td>$v_1^i$</td>
<td>$v_2^i$</td>
</tr>
<tr>
<td>I</td>
<td>1.09</td>
<td>0.74</td>
<td>0.52</td>
</tr>
<tr>
<td>II</td>
<td>1.23</td>
<td>0.75</td>
<td>0.57</td>
</tr>
<tr>
<td>III</td>
<td>1.15</td>
<td>0.73</td>
<td>0.55</td>
</tr>
<tr>
<td>IV</td>
<td>1.16</td>
<td>0.73</td>
<td>0.55</td>
</tr>
</tbody>
</table>

In this table, Ave. and Max. represent the average and maximum; $v^i$, $v_1^i$ and $v_2^i$ (i = I, II, III, IV) represent the velocity of the 286 grids, and the component in the north and east directions, inverted by the i-th method, respectively; $dv$, $dv_1$ and $dv_2$ denote the corresponding differences between the above values of the 286 grids inverted by using Methods I–III, relative to Method IV, separately.
where \( n_1 \) and \( n_2 \) are the numbers of the observed average strain rate tensor data and the GPS velocity field data, respectively. Because there are three average strain rate tensor components in each grid, \( n_1 \) is three times \( N \) in Eq. (4). Similarly, because there are two horizontal velocity components at each GPS site, \( n_2 \) is two times \( M \) in Eq. (4). The solution of Eq. (11) can be expressed as

\[
\hat{\theta} = S^{-1} W_0
\]

In practical application, iteration is needed to obtain the solution by adjusting the weight as follows:

\[
p^k_i = \frac{c}{\hat{\sigma}_{01}^2} p^{k-1}_i, \quad (i = 1, 2)
\]

where \( c \) is an arbitrary positive constant. The iteration does not stop until the variances of unit weight are identical or their ratio can be regarded as 1.0 through some necessary tests, such as hypothesis testing, and so on. The weight scaling factor can be solved by

\[
\lambda = \frac{\prod (c/\hat{\sigma}_{01}^2)}{\prod (c/\hat{\sigma}_{02}^2)}, \quad (i = 1, 2; \ j = 1, 2, 3 \ldots)
\]

where \( (c/\hat{\sigma}_{01}^2) \) denotes the adjustment factor of the weight of the \( i \) th data in the \( j \) th VCE iteration. It controls the variance of \( \lambda \). If the ratio of \( \hat{\sigma}_{01}^2 \) to \( \hat{\sigma}_{02}^2 \) is larger than 1.0, \( \lambda \) decreases, and if the ratio of \( \hat{\sigma}_{01}^2 \) to \( \hat{\sigma}_{02}^2 \) is smaller than 1.0, \( \lambda \) increases. Only when the ratio is nearly 1.0 does \( \lambda \) become stable (the numerator and the denominator in Eq. (14) vary with the same proportion), and all the inversion parameters can be acquired. Eq. (14) expresses the solution of \( \lambda \) through two datasets, and it also can be expanded to more datasets to solve \( \lambda \), only expanding the domain of \( i \) to vary from 1 to the number of types of datasets.

2.4. Method IV: without considering the weight scaling factors

Often, the weight scaling factors are regarded equal, which means that we do not rescale the a priori variance–covariance matrices of the different datasets, so \( \lambda \) is taken as 0.5 in Eq. (4) (e.g. Haines and Holt, 1993; Holt and Haines, 1995; Shen-Tu et al., 1998; Shen-Tu and Holt, 1999; Holt et al., 2000). In this paper, this method is compared with the other three methods to judge their applicability in the geodetic–geophysical joint inversion.

3. Results

While fixing the weight scaling factors by using the above Methods (I–IV), we do the geodetic–geophysical joint inversion from GPS data and seismic moment tensor data, and obtain the crustal motion velocity field and the strain rates in the study area (mainland China and its neighborhood). The spatial distributions of focal mechanisms of earthquakes, the geological background, the irregular grids and GPS sites are concerned in the study area (Fig. 1). In this paper, we plot 286 irregular grids, collect 911 seismic moment tensors from the Harvard centroid moment tensor (CMT) Catalog from 1903 to 2003, and select 1139 GPS velocity observations from the China Earthquake Administration, referred to Eurasian Plate, as the original data for the geodetic–geophysical joint inversion. Fig. 1
also shows that the distribution of GPS sites is heavily nonuniform, dense to the east and sparse to the west.

Table 1 summarizes the results inverted for by these four methods. Included are the weight scaling factor, the value of the objective function, and the a posteriori standard error of unit weight. Then the velocity field and the strain rates in the irregular grids inverted using the weights given by Methods I–IV (Tables 2 and 3) were compared and the corresponding differences in the regular grids ($2^9 \times 2^9$) were shown visually (Figs. 2 and 3).

Table 1 shows that the weight scaling factor is 1.0 for Method I, which indicates that GPS data is useless in the inversion, and only the seismic data is used. The reason for occurring such circumstances is that the inversion of seismic moment tensor data alone leads to the inversion that is barely overdetermined, because of the smoothing for strain rates, and if there is not the smoothing, the inversion would be underdetermined. In contrast, it is common for several GPS velocities to contribute to the estimation of the rotation vector, making the inversion problem mostly overdetermined, except for a few places where data are sparse. As a result, the use of an objective function based purely on fit to the data is inappropriate, and leads to a choice of weights that favor the dataset that is closest to an exactly determined inversion or underdetermined inversion. It is therefore clear that it is not reasonable to do the inversion just by confining the objective function to be a minimum, and that some constraints should be given to inversion parameters according to the actual situation. The scaling factor is 0.016 when it is fixed directly by the a priori standard errors of the seismic data and the GPS data, which are $1.6 \times 10^{-9} \text{ a}^{-1}$ and 1.6 mm/a ($0.25 \times 10^{-9} \text{ a}^{-1}$), respectively. These values can be obtained by calculating the averages of the original errors of these data. It means that the seismic data play a very tiny role in the joint inversion. It is due to the low accuracy of the seismic data and the much higher accuracy of the GPS data. But the good results are not only related to high accuracy, but also the distribution of data. So, it is better to use this method according to the actual problem. Table 1 also shows that the ratios of the a posteriori standard error of the unit weight from each dataset obtained by these methods are 8.816/2.204/0.25, 0.347, 0.996 and 0.780 separately. It means that these two data are merged at last by using VCE because they have the same a posteriori variances. And the weight scaling factor got by using VCE is 0.671, which is closest to 0.5 when compared with the other weight scaling factors. Not only the scaling factor, but also the other items in the table by Methods III and IV are very close. Furthermore, 0.671 means that the seismic data play a slightly more important role in the geodetic–geophysical joint inversion than the GPS data. This is because: (1) the strain rates in each grid are obtained by averaging neighboring grids, so there are enough strain rates in each grid to invert for the unknown parameters, while GPS data is applied as constraints to the velocity; (2) although there are a great number of GPS sites, their spatial distribution is not uniform: more than one GPS site for some grids, but none for others. Thus, the nonuniform distribution of the GPS data reduces their constraints to the velocity, and then reduces the weight scaling factors in the geodetic–geophysical joint inversion. Table 2 shows that the velocity field obtained by fixing the scaling factor when the objective is minimum (Method I), through the a priori information (Method II) and by VCE (Method III) are different from that without considering the scaling factor (Method IV). The maximums of the difference are 0.88 cm/a, 1.33 cm/a and 0.23 cm/a, respectively. It is clearly shown that various differences

Fig. 2. Inverted velocity field in regular grids ($2^9 \times 2^9$) by Methods I–IV. (A)–(C) represent the differences between the former three results and the last one, separately.
Fig. 3. Inverted strain rates in regular grids (20 × 20) by Methods I–IV. (A)–(C) represent the differences between the former three results and the last one, separately.

Fig. 4. The statistical values of differences about strain rates. A, B and C represent the difference got by Method I, II or III, relative to Method IV, separately (according to Table 3).

between Methods III and IV are minimum. Table 3 shows that the strain rates, obtained by Methods I–III are also different from that by Method IV. But various differences between Methods III and IV are also minimum (Fig. 4).

4. Cross validation

In order to test model reliability and appraise the inversion results, we apply a cross validation method (Efron and Tibshirani, 1993, 1997), which is the statistical practice of partitioning a sample data into subsets such that the analysis is initially performed on a single test, while the other subset(s) are retained for subsequent use in confirming and validating the initial analysis. Firstly, from the total 1139 GPS velocity observations, we randomly select 139 and 249 strain rate tensors, mixed with 139 and 249 GPS velocity observations, as the mixed test sets: test set (mix1) and test set (mix2), respectively, and use the rest to do the joint inversion separately. Lastly, we compare the modeled and observed values of the test set through the internal fitting precision and the “external fitting” precision (Table 4).

The internal fitting precision can be expressed by RMS misfit between the modeled and observed velocity (strain rate) components of the inversion set.

\[
\sigma_{IG} = \sqrt{\frac{\sum M_1 (V_{em} - V_{oe})^2 + (V_{en} - V_{oe})^2}{2M_1}}
\]

\[
\sigma_{IS} = \sqrt{\frac{\sum N_1 \left[ \frac{1}{2} (\epsilon_{m} - \epsilon_{o})^2 + \frac{1}{2} \epsilon_{o}^2 \right]}{3N_1}}
\]

where \(\sigma_{IG}\) and \(\sigma_{IS}\) are the internal fitting precisions of GPS data and strain rate data, respectively, subscripts m and o denote the modeled value and observed value separately, \(V_e, V_n\) are the east and north components of velocity, respectively, \(\epsilon_{m}, \epsilon_{o}\) are the horizontal strain rate components, \(M_1\) and \(N_1\) are the number of inversion set for GPS velocity and strain rates separately.
Table 4
The internal fitting precision and the “external fitting” precision

<table>
<thead>
<tr>
<th>Method</th>
<th>$\sigma_{IG}^{139}$ (cm/a)</th>
<th>$\sigma_{EG}^{139}$ (cm/a)</th>
<th>$\sigma_{IG}^{249}$ (cm/a)</th>
<th>$\sigma_{EG}^{249}$ (cm/a)</th>
<th>$\sigma_{IS}^{mix1}$ ($\times 10^{-9}$ a$^{-1}$)</th>
<th>$\sigma_{IS}^{mix2}$ ($\times 10^{-9}$ a$^{-1}$)</th>
<th>$\sigma_{ES}^{mix1}$ ($\times 10^{-9}$ a$^{-1}$)</th>
<th>$\sigma_{ES}^{mix2}$ ($\times 10^{-9}$ a$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.265</td>
<td>0.288</td>
<td>0.271</td>
<td>0.289</td>
<td>4.208</td>
<td>4.337</td>
<td>3.838</td>
<td>3.754</td>
</tr>
<tr>
<td>II</td>
<td>0.222</td>
<td>0.298</td>
<td>0.218</td>
<td>0.301</td>
<td>21.399</td>
<td>22.009</td>
<td>23.131</td>
<td>22.576</td>
</tr>
<tr>
<td>III</td>
<td>0.233</td>
<td>0.270</td>
<td>0.230</td>
<td>0.268</td>
<td>7.329</td>
<td>7.579</td>
<td>7.809</td>
<td>8.075</td>
</tr>
<tr>
<td>IV</td>
<td>0.229</td>
<td>0.270</td>
<td>0.227</td>
<td>0.269</td>
<td>8.312</td>
<td>8.713</td>
<td>8.560</td>
<td>9.033</td>
</tr>
</tbody>
</table>

* In this table, $\sigma_{IG}^{139}$ and $\sigma_{EG}^{139}$ represent the internal fitting precision and the “external fitting” precision of GPS for test set (139/249), respectively; $\sigma_{IS}^{mix1}$ and $\sigma_{ES}^{mix1}$ represent the internal fitting precision and the “external fitting” precision of strain rates for test set (mix1/mix2), respectively.

The “external fitting” precision can be expressed by the RMS between the modeled and observed velocity (strain rate) components of the test set.

$$\sigma_{EG} = \sqrt{\frac{1}{M_2} \sum_{i=1}^{M_2} [(V_{e}^{m} - V_{e}^{o})^2 + (V_{n}^{m} - V_{n}^{o})^2]}$$

(16a)

$$\sigma_{ES} = \sqrt{\frac{1}{3N_2} \sum_{i=1}^{N_2} [(\ddot{\epsilon}_{eg}^{m} - \ddot{\epsilon}_{eg}^{o})^2 + (\ddot{\epsilon}_{en}^{m} - \ddot{\epsilon}_{en}^{o})^2 + (\ddot{\epsilon}_{oo}^{m} - \ddot{\epsilon}_{oo}^{o})^2]}$$

(16b)

where $\sigma_{IG}$ and $\sigma_{ES}$ are the “external fitting” precisions of GPS data and strain rate data, respectively, $M_2$ and $N_2$ are the number of the test set for GPS velocity and strain rates separately.

The internal and external fitting precisions are given in Table 4, and the stability of the method can judge from the difference between these two quantities. The difference is smaller, the method is more stable.

Table 4 shows that the internal fitting precision is slightly higher than the “external fitting” precision in general. Method I gives the best match between the internal and external fitting precision, but it does not mean that this method is the best. As mentioned above, the scaling factor solved by Method I is 1.0, which indicates that the joint inversion can be done by seismic moment tensor data alone, and it has little effect on GPS data, whether GPS data is in the joint inversion can be done by seismic moment tensor data alone, and it has little effect on GPS data, whether GPS data is in the joint inversion can be done by seismic moment tensor data alone, and it has little effect on GPS data, whether GPS data is in the joint inversion can be done by seismic moment tensor data alone, and it has little effect on GPS data. This work was supported by the National Natural Science Foundation of China (No. 40574006, No. 40721001), the Program for New Century Excellent Talents in University (NCET-04-0681), the Programme of Introducing Talents of Discipline to Universities (No. B07037), and Key Laboratory of Dynamic Geodesy, Chinese Academy of Sciences (L04-02). The authors thank Prof. Randell Stephenson, Prof. Paul Cross, Dr. Zhenhong Li and two anonymous for their detailed, constructive comments. The first author would like to thank the Ministry of Science, Research and the Arts Baden Württemberg (Germany) for the fellowship grant.

5. Conclusions

Although the geodetic–geophysical joint inversion relies on the spatial distribution of data to some extents in a joint inversion with different kinds of datasets, the weight of these datasets should be considered in the joint inversion model; the weight scaling factor should be fixed properly.

The Helmert method has been used for many problems, but in the geodetic–geophysical joint inversion it was first introduced to determine weight scaling factors for different datasets. Compared to other current methods for fixing the weight scaling factor, as discussed in the paper, more reasonable results can be obtained by using the Helmert method of variance components estimation.

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